Section 12.2 The Ideal Gas Law

Solutions for Practice Problems

Student Edition page 556

21. Practice Problem (page 556)

What is the volume of 5.65 mol of helium gas at a pressure of 98 kPa and a temperature of 18.0°C?

What Is Required?

You need to find the volume, V, of helium gas, He(g).

What Is Given?

You know the conditions of temperature and pressure:

$T = 18.0°\text{C}$

$P = 98\ \text{kPa}$

You know the amount in moles of the helium gas: $n = 5.65\ \text{mol}$

Plan Your Strategy

Convert the temperature from the Celsius scale to the Kelvin scale.

Use the universal gas constant: $R = 8.314 \dfrac{\text{kPa} \bullet \text{L}}{\text{mol} \bullet \text{K}}$

Use the ideal gas law: $PV = nRT$

Rearrange the equation to isolate the variable V.

Substitute the given data into the equation and solve for V.

Act on Your Strategy

Temperature conversion:

$$\begin{aligned} T &= 18.0°\text{C} + 273.15 \\ &= 291.15\ \text{K} \end{aligned}$$

Isolation of the variable V:

$$PV = nRT$$

$$\frac{\cancel{P}V}{\cancel{P}} = \frac{nRT}{P}$$

$$V = \frac{nRT}{P}$$

Substitution to solve for V:

$$V = \frac{nRT}{P}$$

$$= \frac{5.65\ \cancel{\text{mol}} \times 8.314 \frac{\cancel{\text{kPa}} \bullet \text{L}}{\cancel{\text{mol}} \bullet \cancel{\text{K}}} \times 291.15\ \cancel{\text{K}}}{98\ \cancel{\text{kPa}}}$$

$$= 139.55621\ \text{L}$$

$$= 1.4 \times 10^2\ \text{L}$$

The volume of the helium gas is 1.4×10^2 L.

Check Your Solution

Estimate the amount in moles using rounded values:

$$\frac{\text{volume}}{\text{molar volume}} \approx \frac{140\ \cancel{\text{L}}}{22\ \cancel{\text{L}}/\text{mol}} \approx 6\ \text{mol}$$

This estimate of the amount in moles is close to the given amount. The calculated answer seems reasonable and correctly shows two significant digits.

22. Practice Problem (page 556)

Propane, C_3H_8, is a common gas used to supply energy for barbecue cookers as well as energy requiring appliances in cabins and cottages, and heavy equipment such as the forklift shown in the photograph below. If a tank contains 20.00 kg of propane, what volume of propane gas could be supplied at 22°C and 100.5 kPa?

What Is Required?

You need to find the volume, V, of the propane, $C_3H_8(g)$.

What Is Given?

You know the conditions of temperature and pressure:

T = 22°C

P = 100.5 kPa

You know the mass of the propane gas: m = 20.00 kg

Isolation of the variable V:

$$PV = nRT$$

$$\frac{\cancel{P}V}{\cancel{P}} = \frac{nRT}{P}$$

$$V = \frac{nRT}{P}$$

Substitution to solve for V:

$$V = \frac{nRT}{P}$$

$$= \frac{453.4119\ \cancel{\text{mol}} \times 8.314\ \frac{\cancel{\text{kPa}} \bullet \text{L}}{\cancel{\text{mol}} \bullet \cancel{\text{K}}} \times 295.15\ \cancel{\text{K}}}{100.5\ \cancel{\text{kPa}}}$$

$$= 1.10708 \times 10^4\ \text{L}$$

$$= 1.1 \times 10^4\ \text{L}$$

The volume of the propane gas that could be supplied at 22°C and 100.5 kPa is 1.1×10^4 L.

Check Your Solution

The correct units have been used and the units of mass and temperature have been converted correctly. The large volume is consistent with the application described in the question and the mass of propane that was used. Two significant digits in the answer are consistent with the data given.

23. Practice Problem (page 556)

Find the Celsius temperature of nitrogen gas if a 5.60 g sample occupies 2.40×10^3 mL at 3.00 atm of pressure.

What Is Required?

You need to find the Celsius temperature of a sample of nitrogen gas, N_2(g).

What Is Given?

You know the mass, volume, and pressure of the sample of nitrogen gas:
m = 5.60 g
$V = 2.40 \times 10^3$ mL
P = 3.00 atm

Plan Your Strategy

Convert the volume of the gas from millilitres to litres.
Convert the pressure of the gas from atm to kilopascals.

Determine the molar mass of the sample of N_2(g).

Calculate the amount in moles of N_2(g) using the relationship $n = \frac{m}{M}$.

Use the universal gas constant: $R = 8.314 \frac{\text{kPa} \bullet \text{L}}{\text{mol} \bullet \text{K}}$

Use the ideal gas law: $PV = nRT$

Rearrange the equation to isolate the variable T.

Substitute the given data into the equation and solve for T.

Convert the temperature from the Kelvin scale to the Celsius scale.

Act on Your Strategy

Volume conversion:

$$\begin{aligned} V &= 2.40 \times 10^3 \cancel{\text{mL}} \times 1 \times 10^{-3} \text{ L}/\cancel{\text{mL}} \\ &= 2.40 \text{ L} \end{aligned}$$

Pressure conversion:

$$\begin{aligned} P &= 3.00 \cancel{\text{atm}} \times \frac{101.325 \text{ kPa}}{1.00 \cancel{\text{atm}}} \\ &= 303.975 \text{ kPa} \end{aligned}$$

Molar mass, M, of N_2(g):

$$\begin{aligned} M_{\text{N}_2} &= 2(M_{\text{N}}) \\ &= 2(14.07 \text{ g/mol}) \\ &= 28.14 \text{ g/mol} \end{aligned}$$

Amount in moles, n, of N_2(g):

$$\begin{aligned} n_{\text{N}_2} &= \frac{m}{M} \\ &= \frac{5.6 \cancel{\text{g}}}{28.02 \cancel{\text{g}}/\text{mol}} \\ &= 0.19985 \text{ mol} \end{aligned}$$

Isolation of the variable T:

$$PV = nRT$$

$$\frac{PV}{nR} = \frac{\cancel{nR}T}{\cancel{nR}}$$

$$T = \frac{PV}{nR}$$

Substitution to solve for T:

$$T = \frac{PV}{nR}$$

$$= \frac{303.975\ \cancel{\text{kPa}} \times 2.40\ \cancel{\text{L}}}{0.19985\ \cancel{\text{mol}} \times 8.314 \dfrac{\cancel{\text{kPa}} \bullet \cancel{\text{L}}}{\cancel{\text{mol}} \bullet \text{K}}}$$

$$= 439.0711\ \text{K}$$

Temperature conversion:

$$T = 439.0711\ \text{K} - 273.15$$
$$= 165.921°\text{C}$$
$$= 166°\text{C}$$

The temperature of the nitrogen gas is 166°C.

Check Your Solution

The high pressure of 3.00 atm would decrease the volume. The temperature is high and would increase the volume. The answer seems reasonable and correctly shows three significant digits.

24. Practice Problem (page 556)

What is the pressure of 3.25 mol of hydrogen gas that occupies a volume of 67.5 L at a temperature of 295 K?

What Is Required?

You need to find the pressure of a sample of hydrogen gas, $H_2(g)$.

What Is Given?

You know the amount, volume, and temperature of the sample of hydrogen gas:

n = 3.25 mol
V = 67.5 L
T = 295 K

Plan Your Strategy

Use the universal gas constant: $R = 8.314 \dfrac{\text{kPa} \bullet \text{L}}{\text{mol} \bullet \text{K}}$

Use the ideal gas law: $PV = nRT$

Rearrange the equation to isolate the variable P.

Substitute the given data into the equation and solve for P.

Act on Your Strategy
Isolation of the variable P:

$$PV = nRT$$

$$\frac{P\cancel{V}}{\cancel{V}} = \frac{nRT}{V}$$

$$P = \frac{nRT}{V}$$

Substitution to solve for P:

$$P = \frac{nRT}{V}$$

$$= \frac{3.25\ \cancel{\text{mol}} \times 8.314\ \frac{\text{kPa} \bullet \cancel{\text{L}}}{\cancel{\text{mol}} \bullet \cancel{\text{K}}} \times 295\ \cancel{\text{K}}}{67.5\ \cancel{\text{L}}}$$

$$= 118.0895\ \text{kPa}$$

$$= 118\ \text{kPa}$$

The pressure of the hydrogen gas is 118 kPa.

Check Your Solution
The ideal gas equation has been rearranged correctly and the variables have been substituted into the equation using the correct units. The answer seems reasonable and correctly shows three significant digits.

25. Practice Problem (page 556)
A weather balloon filled with helium gas has a volume of 960 L at 101 kPa and 25°C. What mass of helium was required to fill the balloon?

What Is Required?
You need to find the mass of a sample of helium gas, He(g).

What Is Given?
You know the volume, pressure, and temperature of the sample of helium gas:
V = 960 L
P = 101 kPa
T = 25°C

Plan Your Strategy
Convert the temperature from the Celsius scale to the Kelvin scale.
Use the universal gas constant: $R = 8.314\ \frac{\text{kPa} \bullet \text{L}}{\text{mol} \bullet \text{K}}$

Use the ideal gas law: $PV = nRT$
Rearrange the equation to isolate the variable n.
Substitute the given data into the equation and solve for n.
Determine the molar mass of the He(g) using the atomic mass from the periodic table.
Calculate the mass of the He(g) using the relationship $m = n \times M$.

Act on Your Strategy
Temperature conversion:

$$\begin{aligned} T &= 25°\text{C} + 273.15 \\ &= 298.15 \text{ K} \end{aligned}$$

Isolation of the variable n:

$$\begin{aligned} PV &= nRT \\ \frac{PV}{RT} &= \frac{n\cancel{RT}}{\cancel{RT}} \\ n &= \frac{PV}{RT} \end{aligned}$$

Substitution to solve for n:

$$\begin{aligned} n &= \frac{PV}{RT} \\ &= \frac{101 \cancel{\text{kPa}} \times 960 \cancel{\text{L}}}{8.314 \frac{\cancel{\text{kPa}} \bullet \cancel{\text{L}}}{\text{mol} \bullet \cancel{\text{K}}} \times 298.15 \cancel{\text{K}}} \\ &= 39.1153 \text{ mol} \end{aligned}$$

Molar mass, M, of He(g): 4.00 g/mol (from the periodic table)

Mass, m, of He(g):

$$\begin{aligned} m_{\text{He}} &= n \times M \\ &= 39.1153 \cancel{\text{mol}} \times 4.00 \text{ g}/\cancel{\text{mol}} \\ &= 156.46 \text{ g} \\ &= 160 \text{ g} \end{aligned}$$

The mass of the helium gas required to fill the balloon is 160 g.

Check Your Solution
The ideal gas equation has been correctly rearranged to solve for the amount of moles. The variables are substituted correctly into the equation and the units are correct. The answer seems reasonable and correctly shows two significant digits.

26. Practice Problem (page 556)
Find the molar mass of 6.24 g of an unknown gas that occupies 2.5 L at 18.3°C and 100.5 kPa.

What Is Required?
You need to find the molar mass of a sample of gas.

What Is Given?
You know the volume, pressure, temperature, and mass of the sample of the gas:
$V = 2.5$ L
$P = 100.5$ kPa
$T = 18.3$°C
$m = 6.24$ g

Plan Your Strategy
Convert the temperature from the Celsius scale to the Kelvin scale.
Use the universal gas constant: $R = 8.314 \dfrac{\text{kPa} \bullet \text{L}}{\text{mol} \bullet \text{K}}$
Use the ideal gas law: $PV = nRT$
Rearrange the equation to isolate the variable n.
Substitute the given data into the equation and solve for n.
Calculate the molar mass of the gas using the relationship $M = \dfrac{m}{n}$.

Act on Your Strategy
Temperature conversion:
$$T = 18.3°\text{C} + 273.15$$
$$= 291.45 \text{ K}$$

Isolation of the variable n:
$$PV = nRT$$
$$\frac{PV}{RT} = \frac{n\cancel{RT}}{\cancel{RT}}$$
$$n = \frac{PV}{RT}$$

Substitution to solve for n:

$$n = \frac{PV}{RT}$$

$$= \frac{100.5\ \cancel{\text{kPa}} \times 2.5\ \cancel{\text{L}}}{8.314\ \frac{\cancel{\text{kPa}} \bullet \cancel{\text{L}}}{\text{mol} \bullet \cancel{\text{K}}} \times 291.45\ \cancel{\text{K}}}$$

$$= 0.103688\ \text{mol}$$

Molar mass, M, of the gas:

$$M = \frac{m}{n}$$

$$= \frac{6.24\ \text{g}}{0.103688\ \text{mol}}$$

$$= 60.18054\ \text{g/mol}$$

$$= 6.0 \times 10^1\ \text{g/mol}$$

The molar mass of the gas is 6.0×10^1 g/mol.

Check Your Solution
The ideal gas equation has been correctly rearranged to solve for the amount of moles. The variables are substituted into the equation correctly and the units are correct. The answer seems reasonable and correctly shows two significant digits.

27. Practice Problem (page 556)
A scientist isolates 2.366 g of a gas. The sample occupies a volume of 8.00×10^2 mL at 78.0°C and 103 kPa. Calculate the molar mass of the gas. Is the gas most likely to be bromine, krypton, neon, or fluorine?

What Is Required?
You need to find the molar mass of a sample of gas and determine its possible identity.

What Is Given?
You know the volume, pressure, temperature, and mass of the sample of gas:
$V = 8.00 \times 10^2$ mL
$P = 103$ kPa
$T = 78.0$°C
$m = 2.366$ g

Plan Your Strategy
Convert the temperature from the Celsius scale to the Kelvin scale.
Convert the volume to litres.

Use the universal gas constant: $R = 8.314 \frac{\text{kPa} \bullet \text{L}}{\text{mol} \bullet \text{K}}$

Use the ideal gas law: $PV = nRT$
Rearrange the equation to isolate the variable n.
Substitute the given data into the equation and solve for n.

Calculate the molar mass of the gas using the relationship $M = \frac{m}{n}$.

Act on Your Strategy
Temperature conversion:

$$\begin{aligned} T &= 78.0°\text{C} + 273.15 \\ &= 351.45 \text{ K} \end{aligned}$$

Volume conversion:

$$\begin{aligned} V &= 8.00 \times 10^2 \cancel{\text{mL}} \times 1 \times 10^{-3} \text{ L}/\cancel{\text{mL}} \\ &= 0.800 \text{ L} \end{aligned}$$

Isolation of the variable n:

$$PV = nRT$$

$$\frac{PV}{RT} = \frac{n\cancel{RT}}{\cancel{RT}}$$

$$n = \frac{PV}{RT}$$

Substitution to solve for n:

$$\begin{aligned} n &= \frac{PV}{RT} \\ &= \frac{103 \cancel{\text{kPa}} \times 0.800 \cancel{\text{L}}}{8.314 \frac{\cancel{\text{kPa}} \bullet \cancel{\text{L}}}{\text{mol} \bullet \cancel{\text{K}}} \times 351.45 \cancel{\text{K}}} \\ &= 0.028224 \text{ mol} \end{aligned}$$

Molar mass, M, of the gas:

$$M = \frac{m}{n}$$
$$= \frac{2.366 \text{ g}}{0.028224 \text{ mol}}$$
$$= 83.8293 \text{ g/mol}$$
$$= 83.8 \text{ g/mol}$$

The molar mass of the gas is 83.8 g/mol. The gas is likely krypton.

Check Your Solution
The ideal gas equation has been correctly rearranged to solve for the amount of moles. The variables are substituted correctly into the equation and the units are correct. The answer seems reasonable and correctly shows three significant digits.

28. Practice Problem (page 556)
What is the density of carbon dioxide gas, in grams per litre, at SATP?

What Is Required?
You need to find the density of carbon dioxide gas, CO_2(g), at SATP.

What Is Given?
You know the volume, pressure, and temperature of the carbon dioxide gas:
V = 1.000 L
P = 100.0 kPa
T = 298.15 K
You know that density is the mass per unit volume.

Plan Your Strategy
Calculate the molar mass of the CO_2(g) using the atomic masses from the periodic table.
Use the universal gas constant: $R = 8.314 \frac{\text{kPa} \bullet \text{L}}{\text{mol} \bullet \text{K}}$
Use the ideal gas law: $PV = nRT$
Rearrange the equation to isolate the variable n.
Substitute the given data into the equation and solve for n.
Calculate the mass of the CO_2(g) using the relationship $m = n \times M$.
Calculate the density of the CO_2(g) using the relationship $D = \frac{m}{V}$.

Check Your Solution
The ideal gas equation has been correctly rearranged to solve for the amount of moles. The variables are substituted into the equation correctly and the units are correct. The answer seems reasonable and correctly shows four significant digits.

29. Practice Problem (page 556)
A hydrocarbon gas used for fuel contains the elements carbon and hydrogen in percentages of 82.66 percent and 17.34 percent. Some of the gas, 1.77 g, was trapped in a 750 mL round-bottom flask. The gas was collected at a temperature of 22.1°C and a pressure of 99.7 kPa.
a. Determine the empirical formula for this gas.
b. Calculate the molar mass of the gas.
c. Determine the molecular formula for this gas.

What Is Required?
You need to find
a. the empirical formula for the gas.
b. the molar mass of the gas.
c. the molecular formula for the gas.

What Is Given?
You know the temperature, pressure, volume, and mass of the gas:
$T = 22.1°\text{C}$
$P = 99.7 \text{ kPa}$
$V = 750 \text{ mL}$
$m = 1.77 \text{ g}$
You know that the percentage composition of the gas is 82.66% carbon and 17.34% hydrogen.

Plan Your Strategy
a. empirical formula for the gas
Find the empirical formula using the molar masses of carbon and hydrogen and the percentage compositions.

b. molar mass of the gas
Convert the temperature from the Celsius scale to the Kelvin scale.
Convert the volume to litres.
Use the universal gas constant: $R = 8.314 \dfrac{\text{kPa} \bullet \text{L}}{\text{mol} \bullet \text{K}}$
Use the ideal gas law: $PV = nRT$
Rearrange the equation to isolate the variable n.
Substitute the given data into the equation and solve for n.

Calculate the molar mass of the gas using the relationship $M = \frac{m}{n}$.

c. molecular formula for the gas
Compare the molar mass of the unknown gas with the molar mass of the empirical formula. Multiply the empirical formula by the ratio of the two molar masses to obtain the molecular formula.

Act on Your Strategy
a. empirical formula for the gas
For a 100 g sample:
Mass of 82.66% carbon: 0.8266 × 100 g = 82.66 g
Mass of 17.34% hydrogen: 0.1734 × 100 g = 17.34 g

Amount in moles of each element using the relationship $n = \frac{m}{M}$:

For carbon (M = 12.01 g/mol):

$$\begin{aligned} n_{\mathrm{C}} &= \frac{m}{M} \\ &= \frac{82.66\ \cancel{\mathrm{g}}}{12.01\ \cancel{\mathrm{g}}/\mathrm{mol}} \\ &= 6.88259\ \mathrm{mol} \end{aligned}$$

For hydrogen (M = 1.01 g/mol):

$$\begin{aligned} n_{\mathrm{H}} &= \frac{m}{M} \\ &= \frac{17.34\ \cancel{\mathrm{g}}}{1.01\ \cancel{\mathrm{g}}/\mathrm{mol}} \\ &= 17.16831\ \mathrm{mol} \end{aligned}$$

The simplest mole ratio between the two elements provides the empirical formula.

$\frac{6.88259}{6.88259}$ mol of C : $\frac{17.16831}{6.88259}$ mol of H
1 mol of C : 2.49445 mol H
2 mol of C : 5 mol H

The mole ratio between the carbon and the hydrogen is 2:5. The empirical formula for the gas is C_2H_5.

b. molar mass of the gas
Temperature conversion:

$$T = 22.1°\text{C} + 273.15$$
$$= 295.25 \text{ K}$$

Volume conversion:

$$V = 750\,\cancel{\text{mL}} \times 1 \times 10^{-3}\ \text{L}/\cancel{\text{mL}}$$
$$= 7.5 \times 10^{-1}\ \text{L}$$
$$= 0.75 \text{ L}$$

Isolation of the variable *n*:

$$PV = nRT$$
$$\frac{PV}{RT} = \frac{n\cancel{RT}}{\cancel{RT}}$$
$$n = \frac{PV}{RT}$$

Substitution to solve for *n*:

$$n = \frac{PV}{RT}$$
$$= \frac{99.7\,\cancel{\text{kPa}} \times 0.75\,\cancel{\text{L}}}{8.314\frac{\cancel{\text{kPa}}\bullet\cancel{\text{L}}}{\text{mol}\bullet\cancel{\text{K}}} \times 295.15\,\cancel{\text{K}}}$$
$$= 0.0304618 \text{ mol}$$

Molar mass, *M*, of the gas:

$$M = \frac{m}{n}$$
$$= \frac{1.77 \text{ g}}{0.0304618 \text{ mol}}$$
$$= 58.10556 \text{ g/mol}$$
$$= 58 \text{ g/mol}$$

The molar mass of the unknown gas is 58 g/mol.

c. molecular formula for the gas
Molar mass of the unknown gas: $M = 58$ g/mol

Molar mass, M, of the empirical formula C_2H_5:

$$\begin{aligned} M_{C_2H_5} &= 2M_C + 5M_H \\ &= 2(12.01\text{ g/mol}) + 5(1.01\text{ g/mol}) \\ &= 29.07\text{ g/mol} \end{aligned}$$

Ratio of molar masses: $\dfrac{58\text{ g/mol}}{29.07\text{ g/mol}} = \dfrac{2}{1}$

Since the ratio of the molar masses is 2:1, the molecular formula is $C_2H_5 \times 2 = C_4H_{10}$.

The molecular formula for the unknown gas is C_4H_{10}.

Check Your Solution
The simple integer ratio of the two molar masses makes the molecular formula a reasonable answer. The variables are substituted correctly into the ideal gas law equation and the units are correct. The molar mass is correctly expressed to two significant digits.

30. Practice Problem (page 556)
A 10.0 g sample of an unknown liquid is vaporized at 120.0°C and 5.0 atm. The volume of the vapour is found to be 568.0 mL. The liquid is determined to be made up of 84.2% carbon and 15.8% hydrogen. What is the molecular formula for the liquid?

What Is Required?
You need to find the molecular formula for an unknown liquid.

What Is Given?
You know the temperature, pressure, volume, and mass of the vaporized liquid:
$T = 120.0°C$
$P = 5.0$ atm
$V = 568.0$ mL
$m = 10.0$ g
You also know that the percentage composition of the liquid is 84.2% carbon and 15.8% hydrogen.